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TECHNICAL REPORT

FLUTTER ANALYSIS OF A HYDROFOIL NEAR A FREE SURFACE

By: Peter Crimi and John M. Grace

CAL No. BB-1629-S-3

Prepared for:

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August 1965

CORNELL AERONAUTICAL LABORATORY, INC.
OF CORNELL UNIVERSITY, BUFFALO, N. Y. 14221

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Prepared for
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DAVID TAYLOR MODEL BASIN
WASHINGTON, D.C. 20007

FOREWORD

The work reported herein, performed during the period October 1962 — January 1965, was accomplished by the Cornell Aeronautical Laboratory, Inc. (CAL), Buffalo, New York, for the Fluid Dynamics Branch, Seaworthiness and Fluid Dynamics Division, Hydromechanics Laboratory of the David Taylor Model Basin (DTMB), Washington, D. C. This research is part of the Bureau of Ships Fundamental Hydromechanics Research Program, S-R009 01 01, administered by DTMB and performed under Office of Naval Research Contract Nonr 3578(00). The project monitor for this program at DTMB was, initially, Mr. J. Patton and, later, Mr. W. Souders. Dr. P. Crimi and Mr. J. M. Grace of CAL conducted the study under the direction of Dr. I. C. Statler. Assistance in digital computer programing was provided by Mr. H. Selib.

ABSTRACT

The flutter characteristics of a two-dimensional hydrofoil near a free surface are analyzed, with consideration given to the cases of both one and two degrees of freedom. The flutter equations incorporate lift and moment as determined from an exact linearized potential solution. Free-surface effects have been included without approximation.

Results indicate that for the system having two degrees of freedom, the free surface has very little effect on flutter boundaries, despite the large changes in hydrodynamic loading caused by foil-induced surface waves. When the hydrofoil was constrained to move only in pitch, however, flutter solutions were found where none exist in the absence of the free surface, for practical values of mass ratio and pitch-axis position.

SUMMARY

The study reported herein was directed to analytically investigating the flutter characteristics of a two-dimensional hydrofoil near a free surface. The problem was treated for the hydrofoil having either one or two degrees of freedom. The flutter equations incorporate the lift and moment as determined from the exact linearized potential solution for the flow about a fully wetted hydrofoil oscillating in pitch and/or plunging. The effects of the free surface are incorporated in this solution without approximation.

Because the hydrodynamic loading is a function of two parameters involving flutter speed (i. e., Froude number and reduced frequency), it was necessary to generate flutter boundaries by a method of cross-plotting. This situation is analogous to the one encountered in flutter calculations for compressible flow, in which case loading depends on Mach number as well as reduced frequency.

The hydrodynamic lift and moment are greatly altered by the effects of the foil-induced surface waves when the flow is oscillatory. It was expected, therefore, that a comparison of results with those for infinite depth would reveal large differences in the flutter characteristics. It was found, however, that when the system is free to oscillate in both pitching and plunging, there is very little effect on flutter speed attributable to the free surface. Omission of free-surface effects, even for submergence depths as small as one semichord, would generally produce the correct variations in flutter speed for a wide range of parameter values, with the results usually being somewhat conservative.

The situation was found to be markedly different when the system was constrained to move only in pitch. The results contrast sharply with those for infinite submergence depth. While in the latter case flutter only occurs when the pitch axis is forward of the quarter-chord point, free-surface effects cause flutter, at a relatively low forward speed, for the

pitch axis positioned aft of the quarter-chord and forward of midchord. In the absence of the free surface, fluid density must be much less than that of water for flutter in a single degree of freedom to occur. The results obtained indicate, though, that at finite submergence depths a hydrofoil would flutter in a fluid with the density of water. It is pointed out that flutter of this type could present a problem when a hydrofoil is constrained to move primarily in pitch, as could be the case when it is used as a control surface.

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LIST OF SYMBOLS

a Distance aft of midchord of the pitch axis, in semichords

b Hydrofoil semichord

D Submergence depth, in semichords

F Froude number: $F = U/\sqrt{bg}$

g Acceleration due to gravity

h_0 Complex amplitude of plunging motion, in semichords

k Reduced frequency: $k = \frac{\omega b}{U}$

L Complex amplitude of hydrodynamic lift per unit span, made dimensionless by a factor $(2\pi\rho U^2 b)^{-1}$

M Complex amplitude of hydrodynamic moment per unit span about midchord, positive nose up, made dimensionless by a factor $(\pi\rho U^2 b^2)^{-1}$.

m Hydrofoil mass per unit span

r_α Hydrofoil radius of gyration with respect to the pitch axis, in semichords

U Hydrofoil forward speed

x_α The distance aft of the pitch axis of the hydrofoil center of mass, in semichords

α_0 Complex amplitude of pitching motion

μ Mass ratio: $\mu = \frac{m}{\pi \rho b^2}$

ρ Fluid density

ω Circular frequency of oscillation or flutter frequency

ω_h Uncoupled natural frequency in plunging

ω_α Uncoupled natural frequency in pitching

1. Introduction

The similarity in the design problems of aircraft and hydrofoil boats has understandably led to numerous applications of aeronautical technology in the analysis of hydrofoils. Of particular interest have been the recent investigations of hydroelastic instabilities (see, for example, References 1 and 2) which correspond directly to flutter of aircraft. The hydrofoil flutter problem may be contrasted with that of the airfoil, though, in that the hydrofoil is subjected to the effects of a free surface. The unsteady motions of a hydrofoil cause surface waves which markedly alter the hydrodynamic loadings.

Previous analyses of hydrofoil flutter have either omitted free-surface effects or crudely approximated them, because an exact representation was not available. The study of hydrofoil flutter reported below, however, incorporates the lift and moment as computed from an exact linearized potential solution for the flow due to an oscillating hydrofoil near a free surface. The derivation of this solution is reported in Reference 3.

This study was directed to determining the extent of the influence of the free surface on the hydroelastic characteristics of a two-dimensional hydrofoil having either one or two degrees of freedom (i. e., pitching and/or plunging). No attempt was made to carry out a complete parametric study. Rather, selected system parameters, primarily mass ratio and the ratio of uncoupled natural frequencies, were varied, so as to define flutter boundaries which could be compared with those that would obtain in the absence of a free surface.

In Section 2, the equations of motion are developed and the method used for computing flutter points is outlined. The results of the computations are presented and discussed in Section 3.

2. Development of the Flutter Equations

2.1 Discussion of the Hydrodynamic Forces

As a consequence of the study reported in Reference 3, it is evident that the hydrodynamic lift and moment acting on an oscillating hydrofoil can be significantly affected by the presence of the free surface. The extent of the influence of the free surface can best be seen by reproducing certain of the results of Reference 3. Consider a hydrofoil of infinite span and chord $2b$ oscillating in pitching and plunging, as shown in Figure 1.

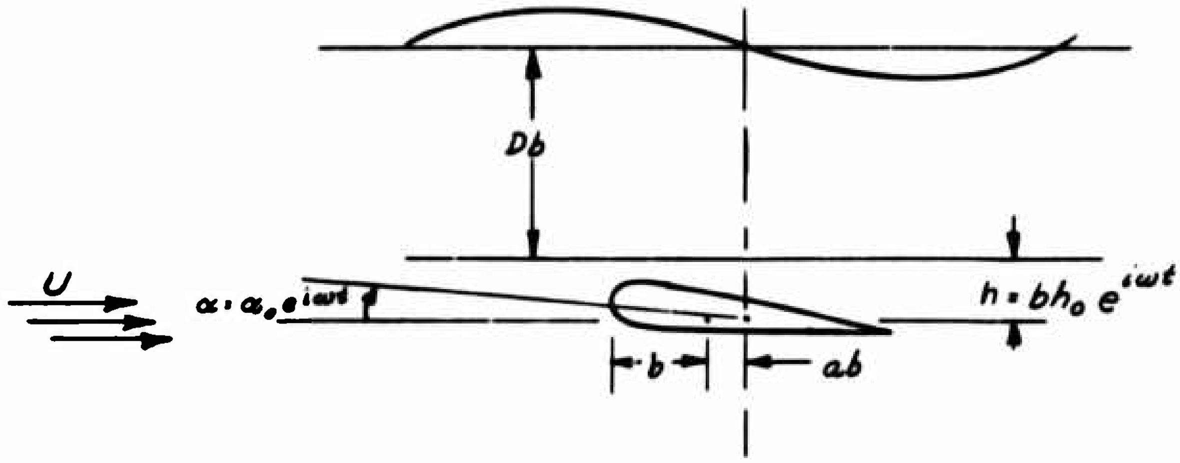

FIGURE 1

Schematic of the System Analyzed

The expressions for the complex amplitudes of the hydrodynamic lift and moment per unit span may be written as follows (see Reference 3), with lift L nondimensionalized by $2\pi\rho U^2 b$ and moment M by $\pi\rho U^2 b^2$:

$$L = -\frac{k^2}{2}\left(h_0 - a\alpha_0\right) + \frac{ik}{2}\alpha_0 + ikH_{1L}(h_0 - a\alpha_0) + H_{2L}\left(1 + \frac{ik}{2}\right)\alpha_0 \tag{1}$$

$$M = \left(-\frac{ik}{2} + \frac{k^2}{8}\right)\alpha_0 + ikH_{1M}(h_0 - a\alpha_0) + H_{2M}\left(1 + \frac{ik}{2}\right)\alpha_0 \tag{2}$$

M is the moment about midchord (positive nose up), h_0, α_0 and a are defined through Figure 1, and $k = \frac{\omega b}{U}$ is the reduced frequency. The functions

H_{1L}, H_{2L}, H_{1M} and H_{2M} depend on k, D and the Froude number $F \equiv \frac{U}{\sqrt{bg}}$. Each of these functions reduces, in the limit of infinite submergence depth, to the classical Theodorsen function. Thus, the deviation of any given function from the Theodorsen result can be considered as a measure of the effect of the free surface on the hydrofoil lift and moment. Since the functional form of each of the functions is quite similar, it is only necessary to consider one example to illustrate the general effect of the free surface. As an example, the real part of H_{1L} is plotted in Figure 2 versus reduced frequency, for a depth of one semichord and a Froude number of unity. The curve is compared with the real part of the infinite-depth (Theodorsen) function in the figure.

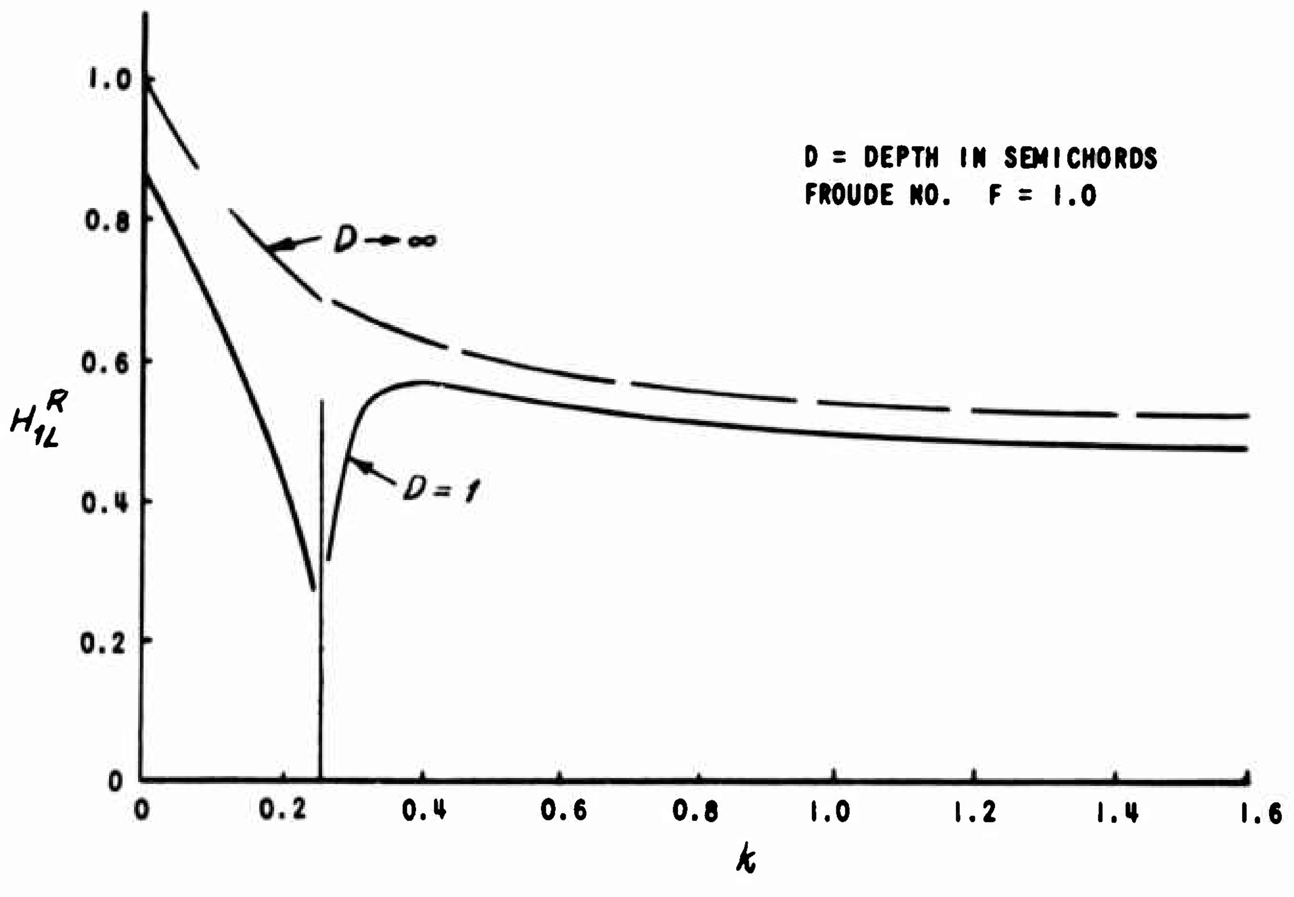

FIGURE 2

Comparison of H_{1L}^R for $F=1$ and $D=1$ with the real part of the Theodorsen function

One can see that the most striking characteristic of this function is its behavior near the so-called resonance condition (i.e., where $k = 1/4F^2$). Physcially, the resonance point can be considered as that point where the group velocity of the wave caused by foil oscillation is equal to the velocity of the foil. The result of this is an increase in the amplitude of the waves with an attendant significant change in the velocity distribution about the foil. The modified velocity distribution, in turn, greatly affects the lift and moment.

The difference in behavior of the hydrodynamic functions for finite and infinite depth is the motivation for this study, since the marked change of these functions near the resonance point could have an appreciable effect on single- and two-degree-of-freedom instabilities. It should be noted that, in the results presented below, flutter data corresponding to values of reduced frequency lower than the resonant frequency are lacking. Generally, there were insufficient lift and moment data to adequately define a flutter boundary for this range of reduced frequencies. However, calculations for selected values of reduced frequencies below resonance produced flutter speeds greater than those calculated for reduced frequencies above resonance.

2.2 Equations of Motion

The equations of motion for a hydrofoil undergoing simple harmonic pitching and/or plunging motions in the absence of structural damping, when nondimensionalized, may be written in the form (see, for example, Reference 4)

$$h_0\mu + \mu x_\alpha \alpha_0 - h_0\mu\left(\frac{\omega_h}{\omega}\right)^2 - \frac{2L}{k^2} = 0 \tag{3}$$

$$h_0\mu x_\alpha + \alpha_0\mu r_\alpha^2 - \alpha_0\mu r_\alpha^2\left(\frac{\omega_\alpha}{\omega}\right)^2 + \frac{1}{k^2}(M + 2aL) = 0 \tag{4}$$

In the above equations, μ denotes the mass ratio $(\mu \equiv m/\pi\rho b^2)$, $x_\alpha b$ the distance of the center of mass aft of the elastic axis, $r_\alpha b$ the radius of gyration about the elastic axis, ω_h the uncoupled natural bending frequency, and ω_α the uncoupled natural torsion frequency. Combination of equations

(3) and (4) above with (1) and (2) of Section 2. 1 yields the following form for the equations of motion:

$$h_0\left[\left(\mu\left(1-(\omega_h/\omega)^2\right)-2\beta_3\right)-2i\beta_4\right]+\alpha_0\left[\left(\mu x_\alpha-2\delta_3\right)-2i\delta_4\right]=0 \tag{5}$$

$$h_0\left[(\mu x_\alpha+\beta_1)+i\beta_2\right]+\alpha_0\left[\left(\mu r_\alpha^2(1-(\omega_\alpha/\omega)^2+\delta_1\right)+i\delta_2\right]=0. \tag{6}$$

The quantities β_1 , β_2 , β_3 , β_4 and δ_1 , δ_2 , δ_3 , δ_4 are defined as:

$$\beta_1 = -\left\{\frac{H_{1M}^I}{k}+a+\frac{2aH_{1L}^I}{k}\right\}$$

$$\beta_2 = \left\{\frac{H_{1M}^R}{k}+\frac{2aH_{1L}^R}{k}\right\}$$

$$\beta_3 = -\left\{\frac{1}{2}+\frac{H_{1L}^I}{k}\right\}$$

$$\beta_4 = \frac{H_{1L}^R}{k}$$

$$\delta_1 = \left\{a\left[\frac{H_{1M}^I}{k}+\frac{2H_{2L}^R}{k^2}-\frac{H_{2L}^I}{k}\right]+a^2\left[1+\frac{2H_{1L}^I}{k}\right]+\frac{H_{2M}^R}{k^2}-\frac{H_{2M}^I}{2k}+\frac{1}{8}\right\}$$

$$\delta_2 = \left\{a\left[\frac{1}{k}+\frac{2H_{2L}^I}{k^2}+\frac{H_{2L}^R}{k}-\frac{H_{1M}^R}{k}\right]-\frac{2a^2}{k}H_{1L}^R+\frac{1}{2k}+\frac{H_{2M}^I}{k^2}\right\}$$

$$\delta_3 = \left\{a\left(\frac{1}{2}+\frac{H_{1L}^I}{k}\right)+\frac{H_{2L}^R}{k^2}-\frac{H_{2L}^I}{2k}\right\}$$

$$\delta_4 = \left\{-a\frac{H_{1L}^R}{k^2}+\frac{1}{2k}+\frac{H_{2L}^I}{k^2}+\frac{H_{2L}^R}{2k}\right\}.$$

The superscripts (R) and (I) denote real and imaginary parts, respectively.

If Equations (5) and (6) are to have a nontrivial solution, the determinant of the coefficients of h_0 and α_0 must vanish. This requirement provides two relations which must be satisfied at a flutter point, since the real and imaginary parts of the (complex) determinant must each equal zero. The method used for obtaining solutions to these two equations is discussed below.

2. 3 Method of Solution of Flutter Equations

2. 3. 1 Two-Degree-of-Freedom System

The method of solution used in this report for solving the flutter equations in the case of two degrees of freedom consisted, first, of specifying the system parameters D, F, k, μ, x_α, r_α and a and solving for the values of (ω_h/ω) and (ω_α/ω) that would satisfy the equations in question. It should be noted that the specification of D, F, and k was sufficient to determine the hydrodynamic lift and moment. The numerical calculations were performed on an IBM 7044 computer. The computer program used is described in Appendix A Section (a).

The second step used in generating appropriate flutter boundaries was to plot an intermediate graph of the form shown in Figure 3, where the ordinate is given by $\omega_\alpha\sqrt{\frac{b}{g}} = Fk\frac{\omega_\alpha}{\omega}$. It is necessary to obtain, first, this intermediate plot rather than the usual plot of $U/b\omega_\alpha$ vs. ω_h/ω_α from the solutions ω_α/ω, ω_h/ω, and the input data, since the latter plot would consist of a family of curves with Froude number as the parameter. These curves could not be regarded as flutter boundaries, since both the ordinate and the parameter, Froude number, are functions of velocity. The purpose of using the intermediate plot with ordinate $\omega_\alpha\sqrt{\frac{b}{g}}$ is to remove this difficulty. The situation is analogous to the one encountered in the flutter analysis of an airfoil in compressible flow, where loading is a function of reduced frequency and Mach number. The final form of flutter boundary is generated from Figure 3 by constructing a cross-plot of $U/b\omega_\alpha$ vs. ω_h/ω_α for constant values of $\omega_\alpha\sqrt{\frac{b}{g}}$ where the ordinate is given by

$$\frac{U}{b\omega_\alpha} = \frac{F}{\omega_\alpha\sqrt{b/g}}.$$

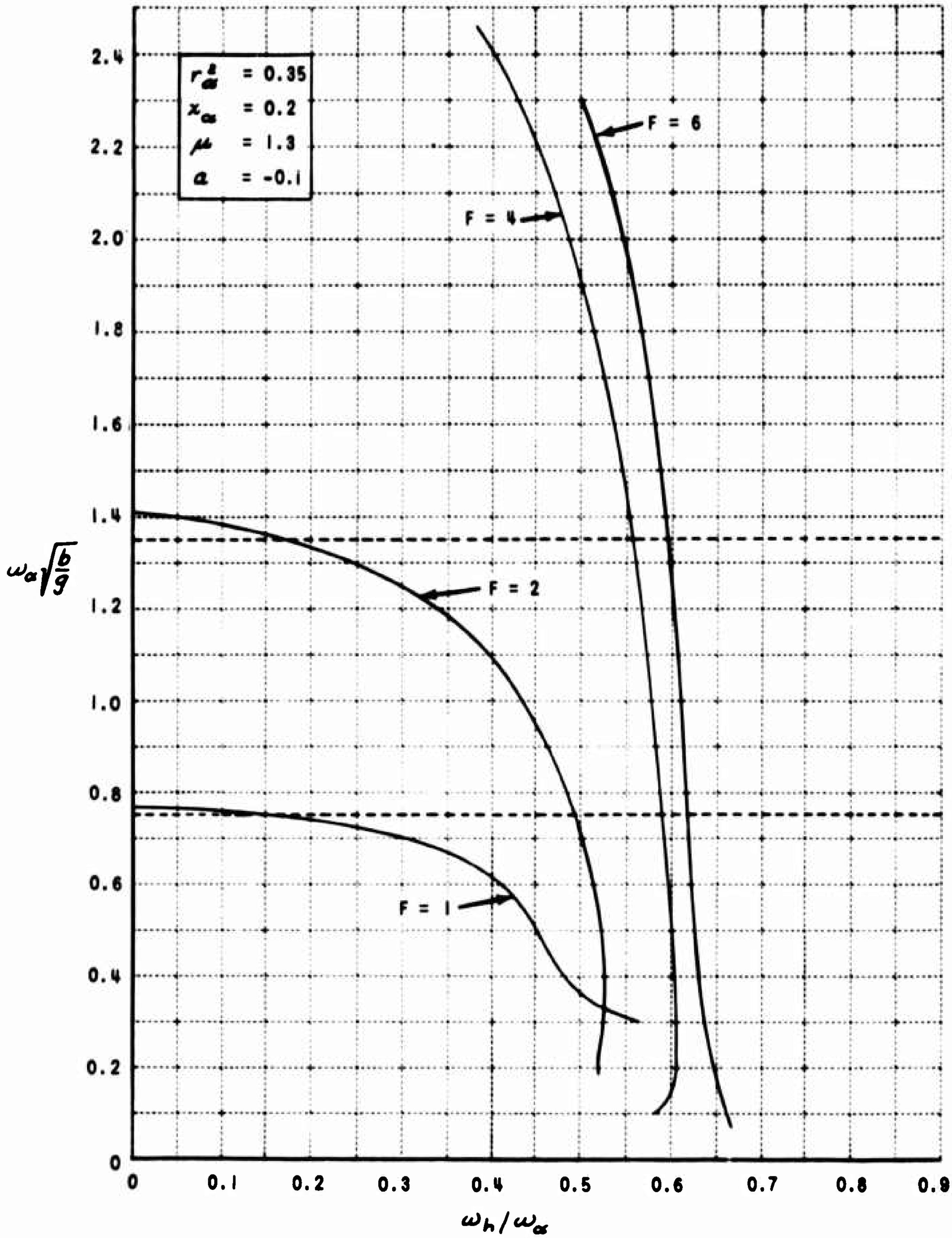

Figure 3 EXAMPLE OF PLOTS USED TO GENERATE FLUTTER BOUNDARIES FOR THE SYSTEM HAVING TWO DEGREES OF FREEDOM

Flutter boundaries with μ as an independent variable were also constructed. These were generated by using an intermediate graph similar to Figure 3, except that, for this case, Froude number is held constant and μ is the parameter of the family of curves. Data from a series of these graphs, each for a constant Froude number, were then obtained for a given value of ω_h/ω_α, thereby allowing a plot of $\frac{U}{b\omega_\alpha}$ vs. μ to be generated.

2.3.2 Single-Degree-of-Freedom System

The flutter equations for a single-degree-of-freedom system can be deduced from Equations (5) and (6). For example, a foil which has infinite stiffness in torsion has the following flutter equations:

$$\mu\left[1-(\omega_h/\omega)^2\right]+\frac{1}{2}+\frac{H_{1L}^I}{k} = 0 \tag{7a}$$

$$\frac{H_{1L}^R}{k} = 0. \tag{7b}$$

For the case in which there is infinite stiffness in the bending mode, Equation (6) gives

$$\mu r_\alpha^2\left[1-(\omega_\alpha/\omega)^2\right]+\delta_1 = 0 \tag{8a}$$

$$\delta_2 = 0. \tag{8b}$$

Since only flutter in torsion was treated, comments concerning the method of solution refer to Equation (8). However, the methods of solution of Equations (7) and (8) are analogous. The computer program used to solve Equation (8a) and (8b) are described in Appendix A Section (b).

As noted previously, the specification of the parameters D , F , and k are sufficient to determine the hydrodynamic functions. Therefore, Equation (8b) may be solved for a once D , F , and k are specified. Equation (8a) may then be solved for (ω_α/ω) using the value of a calculated from (8b) and the specified values of μr_α^2 . The solution for (ω_α/ω) can be written as

$$\left(\frac{\omega_\alpha}{\omega}\right) = \left[1+\frac{\delta_1}{\mu r_\alpha^2}\right]^{1/2}. \tag{9}$$

A plot of $\omega_\alpha\sqrt{\frac{b}{g}}$ vs. a can then be generated, where the ordinate $\omega_\alpha\sqrt{\frac{b}{g}}$ is expressible, from Equation (9), in the form

$$\omega_\alpha\sqrt{\frac{b}{g}} = Fk\left[1+\frac{\delta_1}{\mu r_\alpha^2}\right]^{1/2} . \tag{10}$$

It was found that a more complete definition of flutter boundaries could be obtained with the hydrodynamic data available if μr_α^2, rather than a, were made the independent variable. Thus, intermediate plots of $\omega_\alpha\sqrt{\frac{b}{g}}$ vs. a were first formed, for specified values of D and F, with μr_α^2 as parameter. As an example, one of these intermediate plots is shown in Figure 4 for $D=1$ and $F=1$. Next, cross-plots of $\omega_\alpha\sqrt{\frac{b}{g}}$ vs. μr_α^2, for a specified value of a, were constructed, with Froude number as parameter. Finally, values of $\omega_\alpha\sqrt{\frac{b}{g}}$ were selected and the curves of $U/b\omega_\alpha$ vs. μr_α^2 generated from the latter plots.

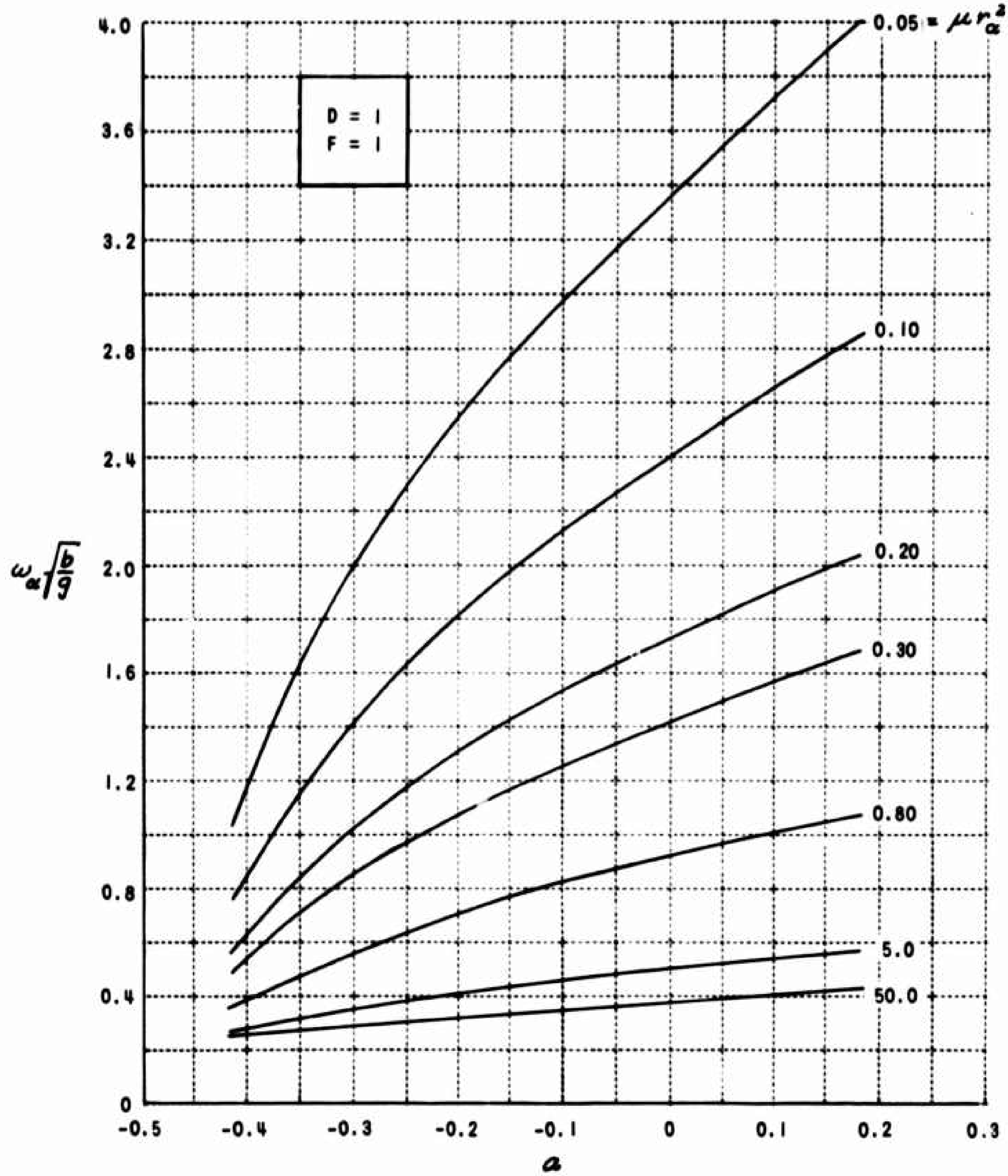

Figure 4 EXAMPLE OF PLOTS USED TO GENERATE FLUTTER BOUNDARIES FOR THE SYSTEM HAVING ONE DEGREE OF FREEDOM

3. Discussion of Results

3.1 Flutter in Two Degrees of Freedom

The computations with both pitching and plunging degrees of freedom included were directed primarily to obtaining an indication of the magnitude of free-surface effects, rather than to generating a complete parametric study. Thus, submergence depth was maintained at one semi-chord, while a single value of 0.2 for mass imbalance x_α was used and r_α^2 was kept at a value of 0.35.

In Figures 5, 6 and 7, dimensionless flutter speed $U/b\omega_\alpha$ is shown plotted against ω_h/ω_α for values of a of -0.1, -0.2 and -0.3, respectively. On each figure, the flutter boundaries for $\omega_\alpha\sqrt{\frac{b}{g}}$ of 0.75 and 1.35 are shown, together with the curve for infinite submergence depth. The value of μ is 1.3 for all these boundaries.

Most surprising about these results is the similarity between the curves for finite and infinite submergence depths. Figure 5 shows no appreciable effect of the free surface on flutter speed. Figures 6 and 7 indicate that, if the pitch axis is moved forward, the free surface has some stabilizing effect, but the shift in the curves can hardly be termed substantial. It might be reasoned that the surface waves which are generated would tend to stabilize the system by carrying off energy, but the effects would be expected to be much larger because the hydrodynamic loading is so greatly affected by the free surface.

In Figures 8, 9 and 10 are shown flutter boundaries, obtained by cross-plotting, with μ as abscissa, for ω_h/ω_α of 0.2, 0.5 and 0.75, respectively. Again, each figure shows the curves for $\omega_\alpha\sqrt{\frac{b}{g}}$ of 0.75 and 1.35 together with the one for infinite depth. Pitch-axis position a is -0.2 for all three of these figures. The free surface is again seen to have very little effect on the flutter speed. As in the previous plots with ω_h/ω_α as independent variable, the surface waves have some stabilizing influence, particularly for $\omega_\alpha\sqrt{\frac{b}{g}} = 1.35$ and for the two smallest values of ω_h/ω_α.

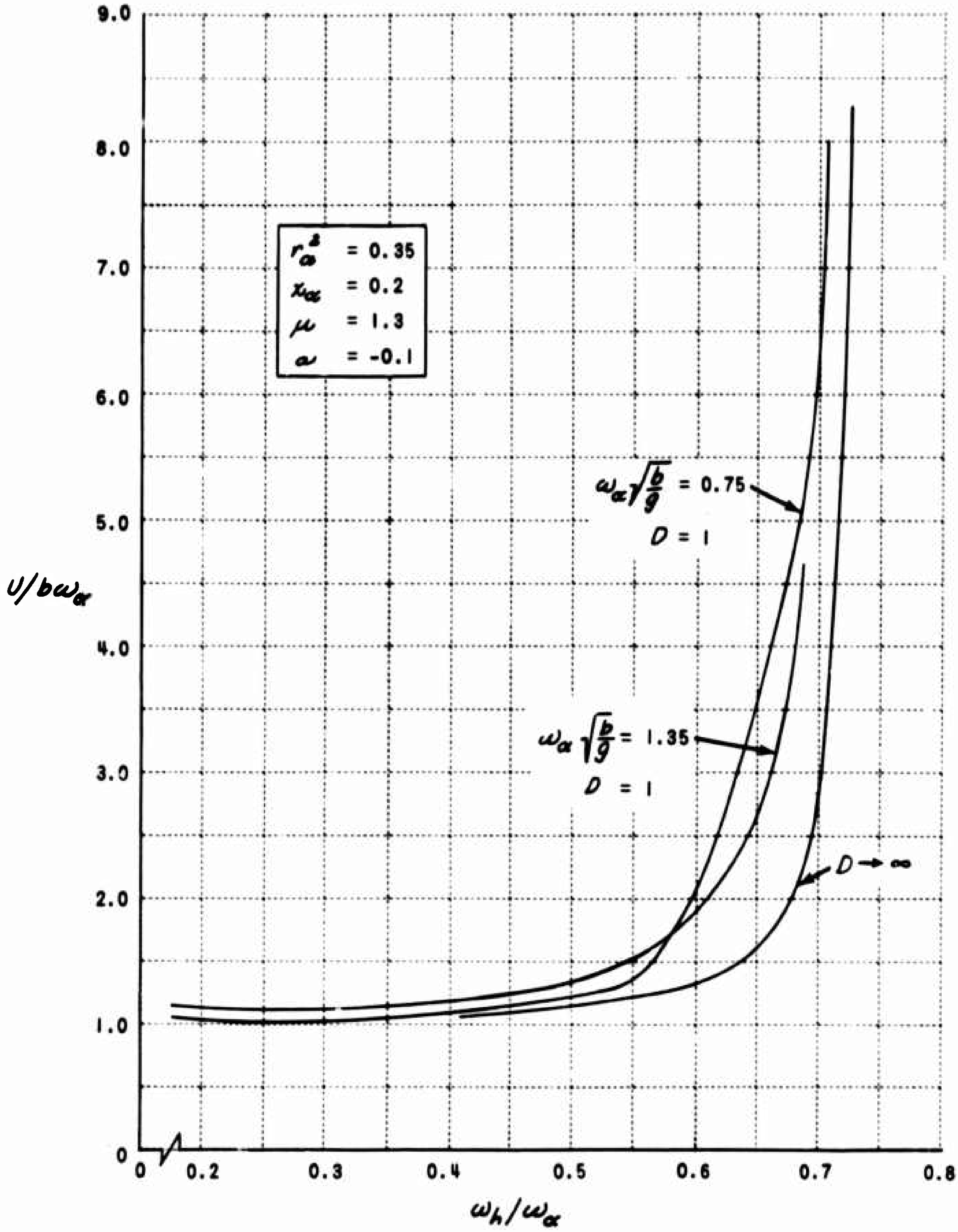

Figure 5 $U/b\omega_\alpha$ vs. ω_h/ω_α for a = -0.1

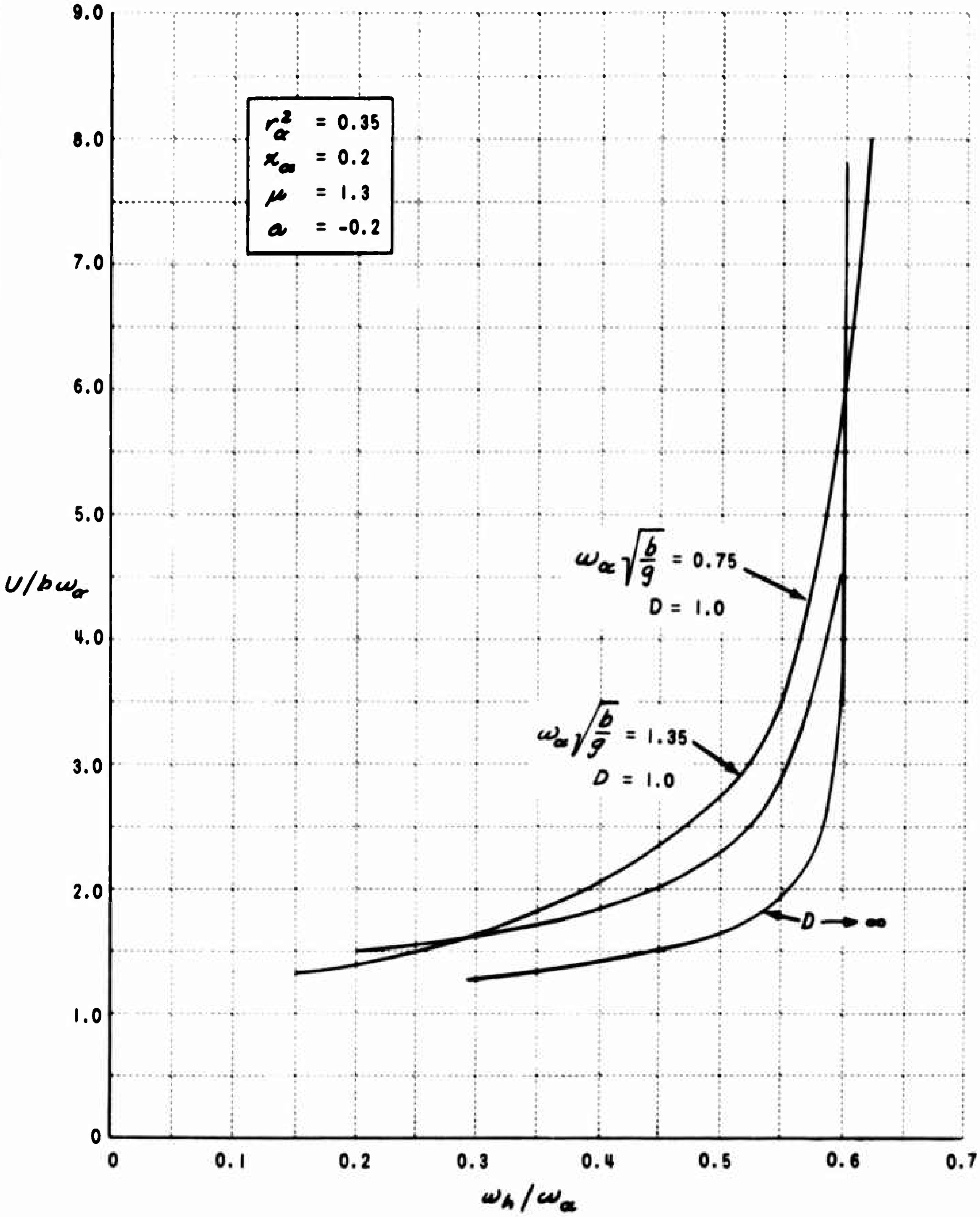

Figure 6 $U/b\omega_\alpha$ vs. ω_h/ω_α for a = -0.2

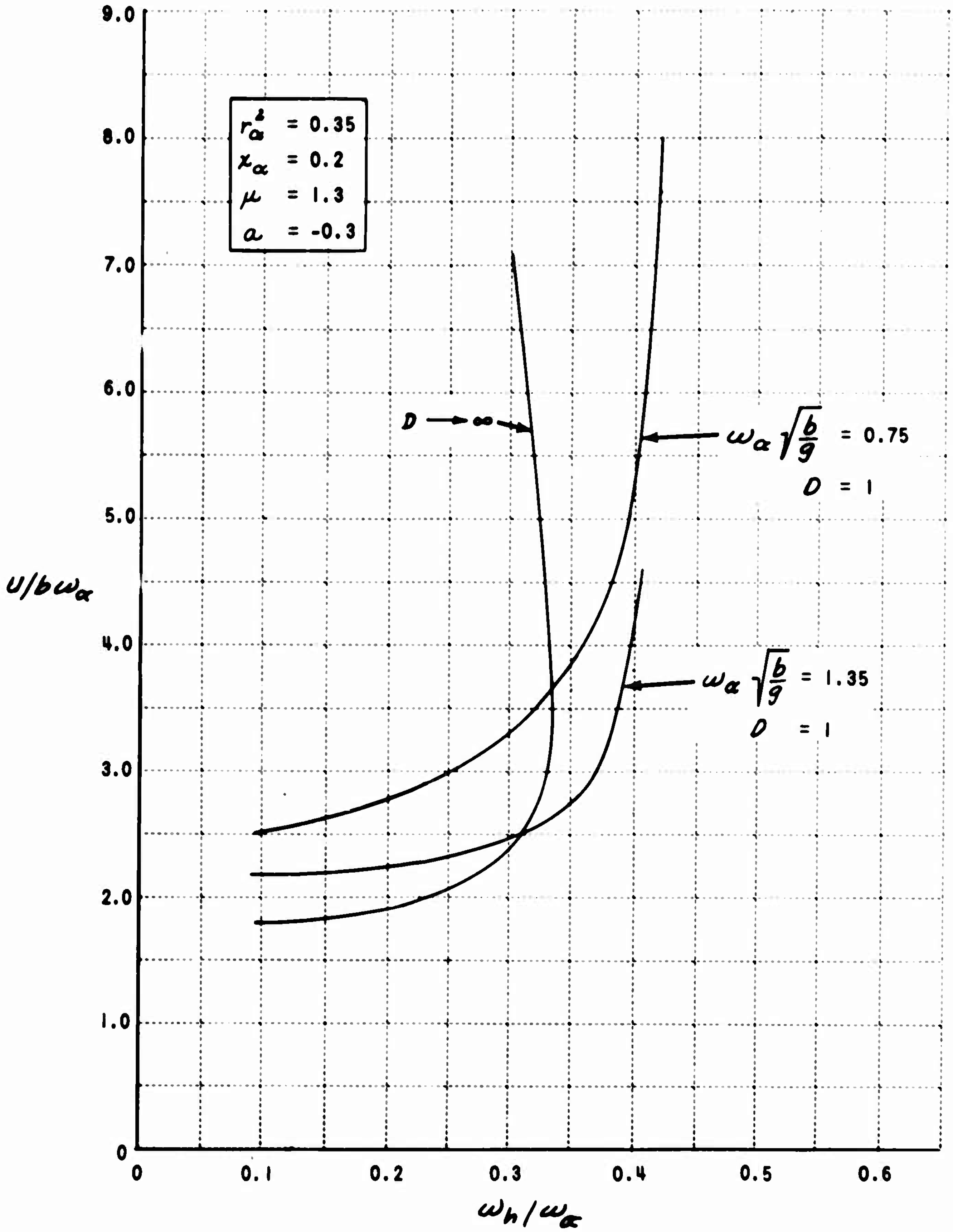

Figure 7 $U/b\omega_\alpha$ vs. ω_h/ω_α for $a = -0.3$

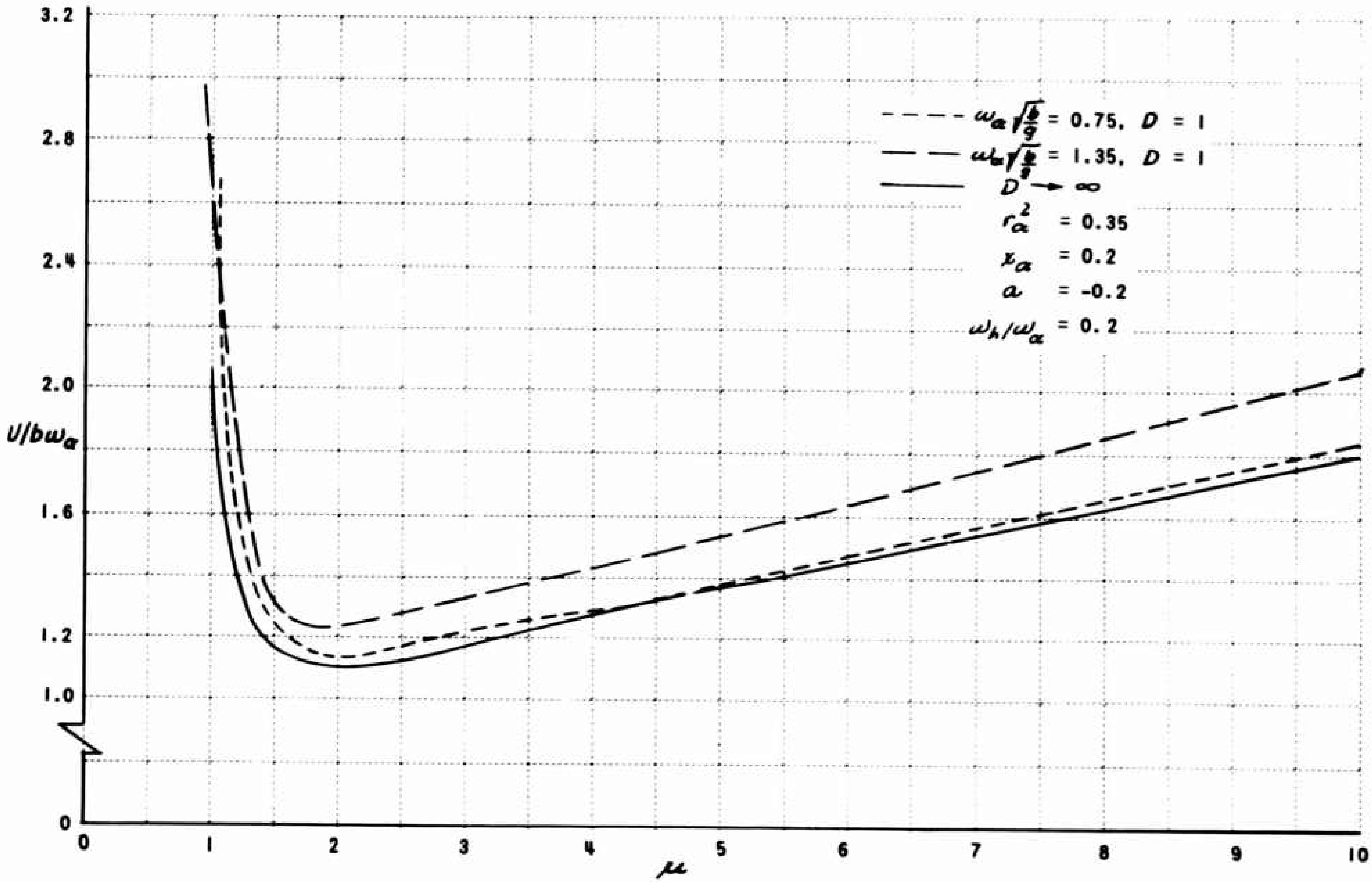

Figure 8 $U/b\omega_\alpha$ vs. μ for $\omega_h/\omega_\alpha = 0.2$

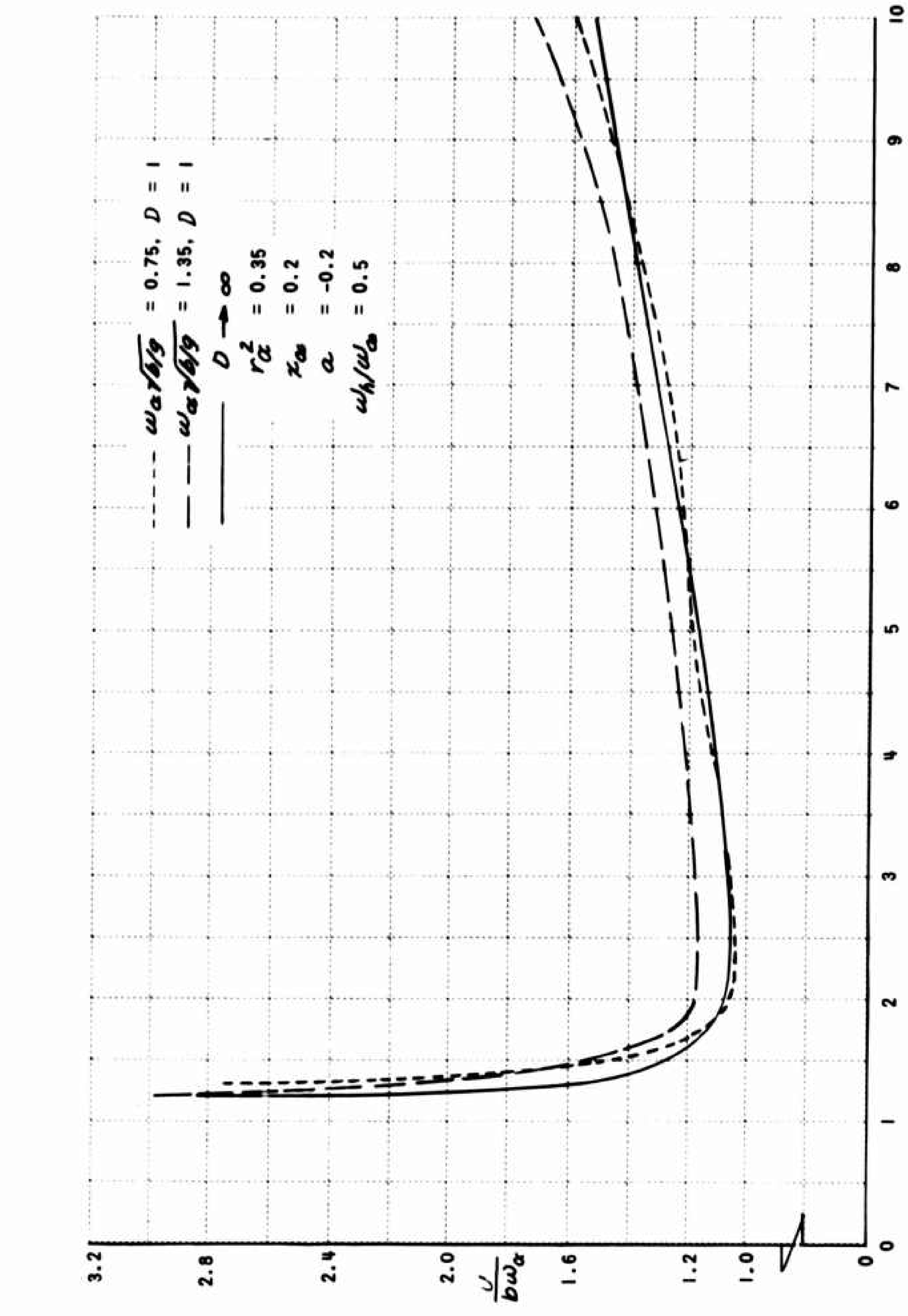

Figure 9 $U/b\omega_\alpha$ vs. μ for $\omega_h/\omega_\alpha = 0.5$

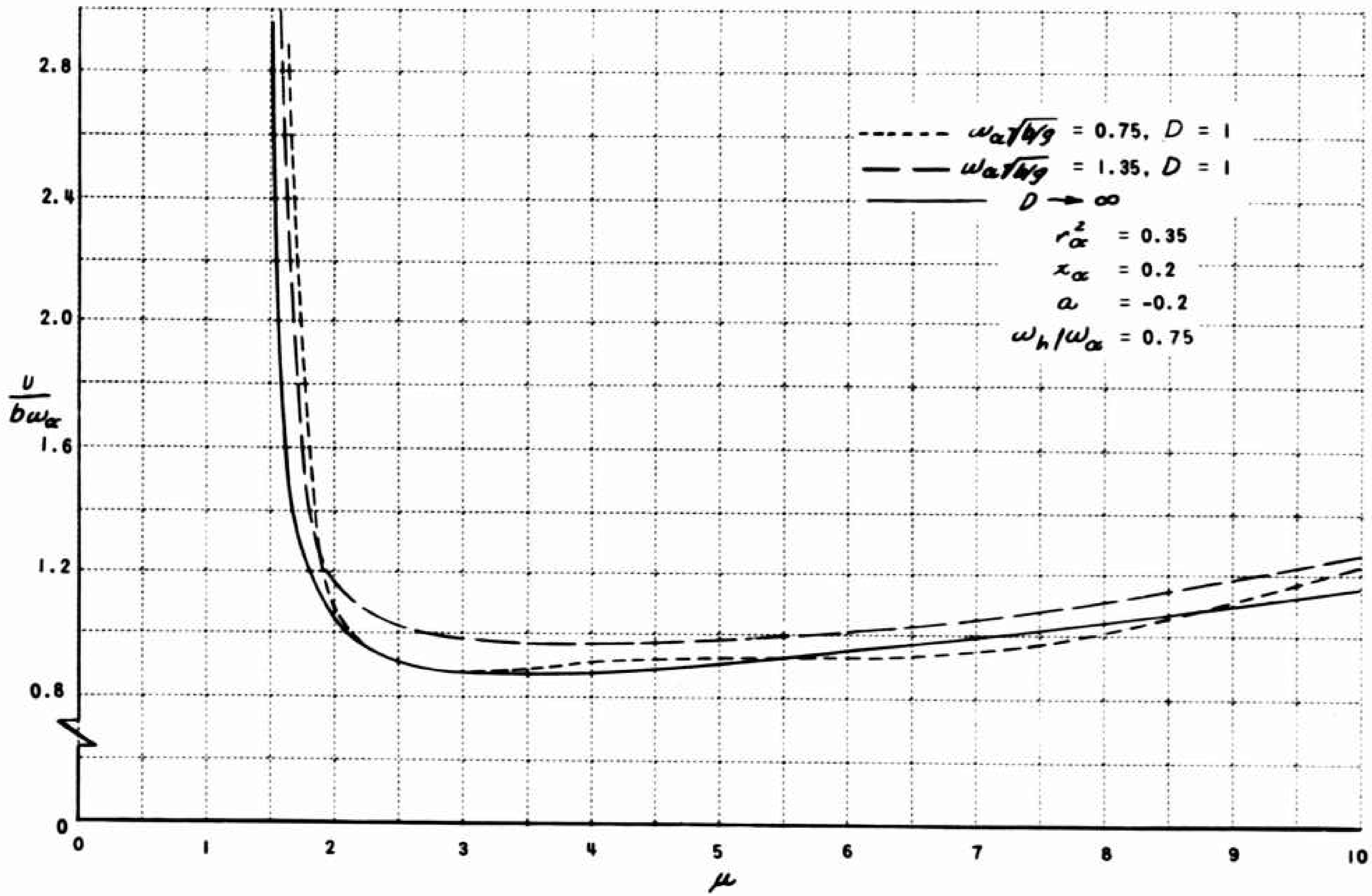

Figure 10 $U/b\omega_\alpha$ vs. μ for ω_h/ω_α = 0.75

It would appear, then, that despite the large effect of the free surface on the hydrodynamic loading, the flutter of hydrofoils having two degrees of freedom can be adequately predicted without taking account of the free surface for submergence depths as small as one semichord. Omission of free-surface effects would generally provide conservative results, and the over-all effects of various system parameters would be correctly obtained.

3.2 Flutter in a Single Degree of Freedom

In contrast with the situation for two degrees of freedom, the dominance of free-surface effects in the hydrodynamic loading was found to greatly alter the flutter characteristics of a hydrofoil with just one degree of freedom. A system of the type being analyzed here, in the absence of free-surface effects, is extremely resistent to flutter when one of the two degrees of freedom is removed. A two-dimensional hydrofoil at infinite submergence depth will not flutter at all if motion is allowed only in plunging, and if it is just free to pitch, flutter only occurs when the pitch axis is ahead of the quarter-chord point (i.e., $a < \frac{-1}{2}$) and the mass ratio μ is much greater than would be encountered by a hydrofoil (see, for example, Reference 5). The addition of free-surface effects was found to alter this situation markedly.

Consideration was limited to flutter in pitch. It appears that flutter in plunging is possible, but it would occur over a very restricted range of reduced frequencies for which lift and moment data were not available. The results of the computations are shown in Figures 11 and 12, where the dimensionless flutter speed $U/b\omega_\alpha$ is plotted against μr_α^2 for selected values of pitch-axis position and $\omega_\alpha\sqrt{\frac{b}{g}}$. The values for a selected, i.e., -.112, -.201 and -.266, were not chosen completely arbitrarily; judicious choices for the values of a greatly facilitated the definition of flutter boundaries because of limitations in the amount of hydrodynamic data available. Again, computations were only performed for a depth of one semichord. It appears, however, that flutter characteristics similar to those obtained would be evident at other finite depths.

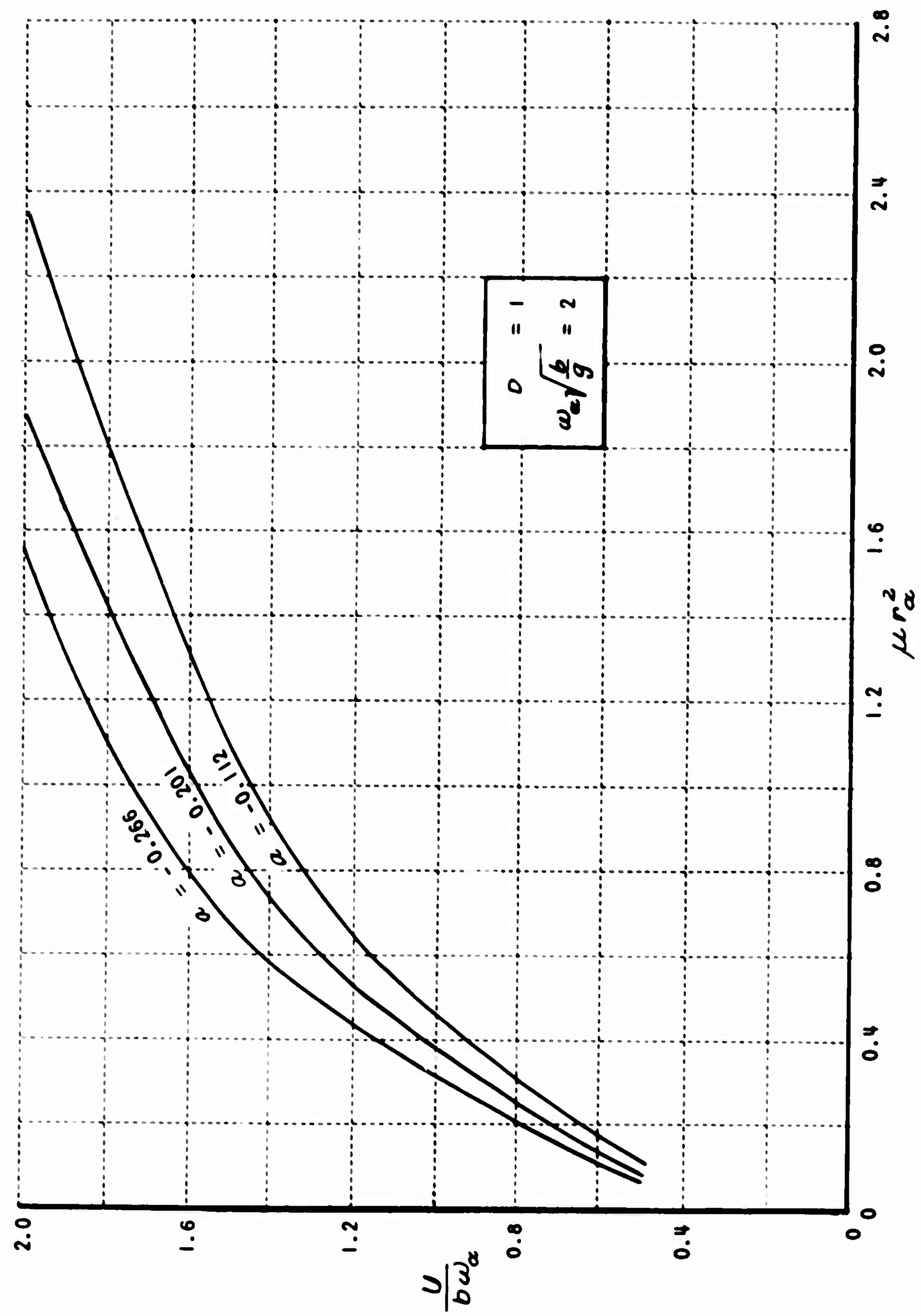

Figure 11 $U/b\omega_\alpha$ vs. μr_α^2 for $\omega_\alpha\sqrt{\frac{b}{g}} = 2$

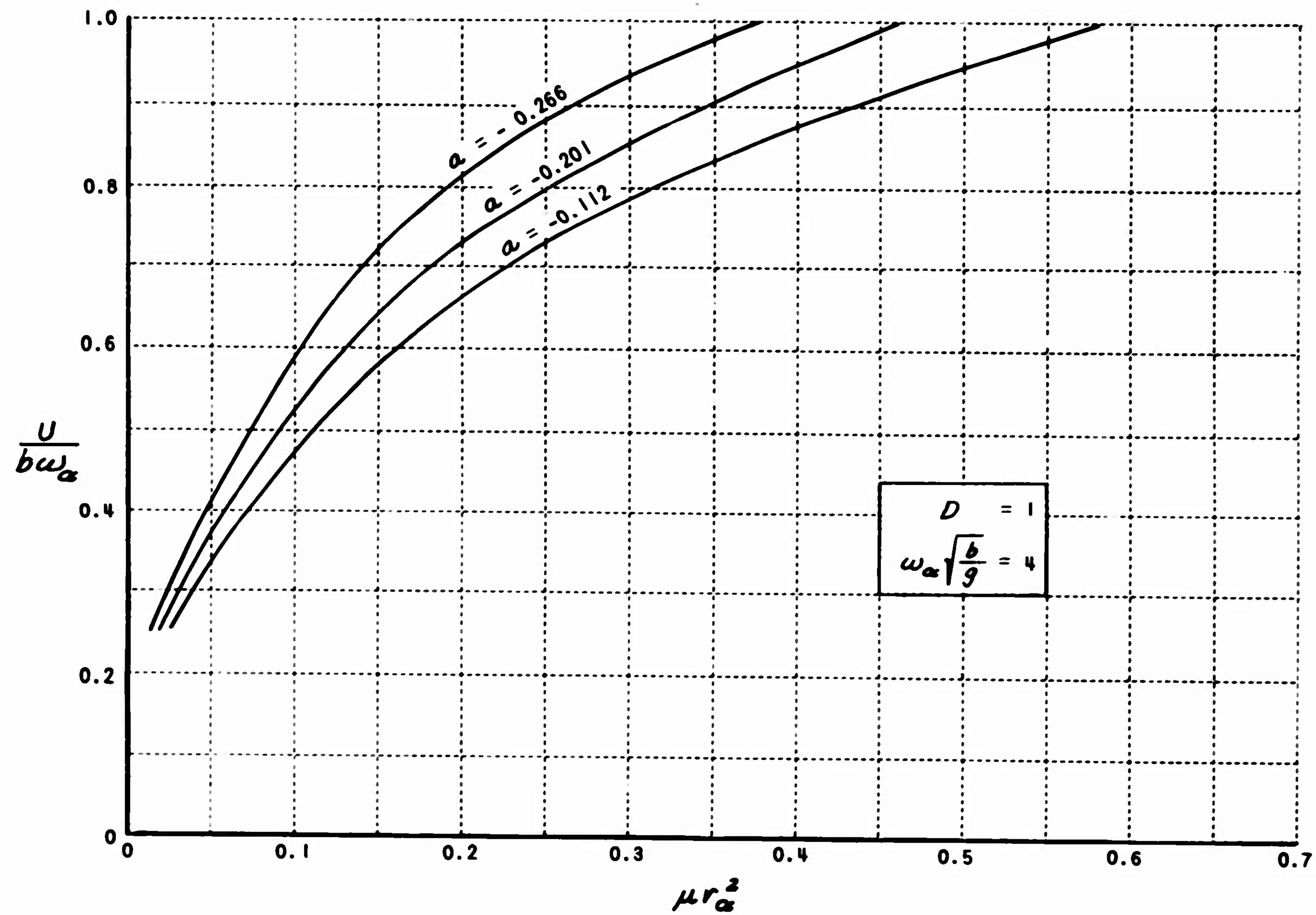

Figure 12 $U/b\omega_\alpha$ vs. μr_α^2 for $\omega_\alpha\sqrt{\frac{b}{g}} = 4$

The results of Figures 11 and 12 indicate that the effects of a free surface are extremely destabilizing for flutter in pitch alone. At finite submergence depth, flutter occurs when the pitch axis is aft, rather than forward, of the quarter-chord point. Instead of there being a minimum in μr_α^2 below which no flutter occurs, as is the case for $D \to \infty$, the system becomes progressively less stable as μr_α^2 decreases. Generally, the reduced frequencies for flutter were found to lie quite near the critical values. This unusual flutter instability may therefore be attributed to the singular behavior of the hydrodynamic loading near the resonance condition.

Instabilities of this type could conceivably be significant in some hydrofoil applications. For example, a hydrofoil used as a control surface could have a pitch frequency, as determined by the stiffness of the control linkages, which is much less than its bending frequency. The foil would then effectively have only a pitching degree of freedom and, so, would be subject to the type of flutter predicted here.

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APPENDIX A

DESCRIPTION OF THE COMPUTER PROGRAM

(A) "Hydrofoil Flutter Program 2 Degrees of Freedom"
Written by H. Selib at the Cornell Aeronautical Laboratory, Inc.

The purpose of the IBM 7044 Fortran IV program generated was to obtain a solution to the equations of motion (3) and (4) of Section 2.2. These equations were solved to obtain the frequency ratio (ω_h/ω_α) for given values of the input parameters namely, D, k, F, a, μ, r_α^2, x_α, g_α, g_h, and the hydrodynamic functions H_{1L}, H_{2L}, H_{1M}, H_{2M}, as defined in Reference 3.

As noted above the program was written in Fortran IV and consists of a main program of length $_8$ 2330 (number of octal locations).

The inputs are as follows:

CARD	FORMAT	
1	I6,F8.5	NC (number of cases for one set of D, F, k, a values); a
2	9F8.5	D, F, k
3	9F8.5	RH_{1L}, IH_{1L}; RH_{2L}, IH_{2L}; RH_{1M}, IH_{1M}; RH_{2M}; IH_{2M}
4	9F8.5	up to Card NC + 4: μ, x_α, r_α^2, g_α, g_h

The program print out for the case of two degrees of freedom flutter consists of values of $\frac{\omega_\alpha}{\omega}$, $\frac{\omega_h}{\omega}$ and products of these quantities with F and k which are of interest. In addition quantitites L_1, L_2, L_3, L_4, M_1, M_2, M_3, M_4 are included in the print out. These quantities can be related to quantities defined on page 5 of this report as follows:

$$L_1 \equiv -2\beta_3 \qquad M_1 \equiv \beta_1$$

$$L_2 \equiv -2\beta_4 \qquad M_2 \equiv \beta_2$$

$$L_3 \equiv -2\delta_3 \qquad M_3 \equiv \delta_1$$

$$L_4 \equiv -2\delta_4 \qquad M_4 \equiv \delta_2$$

The program listing is contained on the following pages.

```
$IBFTC ZZFLTR  LIST,REF
C      HYDROFOIL FLUTTER PROGRAM - TWO DEGREES OF FREEDOM
C      REVISED - 5/20/63
       DIMENSION AAMU(100),XXA(100),RRA(100),GGA(100),GGH(100)
       COMMON AAMU,XXA,RRA,GGA,GGH
       COMMON D,F,AK,A,NC,R1L,XI1L,R1M,XI1M,R2L,XI2L,R2M,XI2M,XL1,XL2,
      1       XL3,XL4,XM1,XM2,XM3,XM4,AMU,XA,RA,GA,GH,V1,V2,AK1,AK2,AK3,
      2       AK4,C1,C2,C3,C4,B0,B1,B2,DISC,LL,SQD,SIG1,SIG2,SIH1,SIH2,
      3       WA12,WA22,WH12,WH22,WAW1,WAW2,WHWA1,WHWA2,VBWA1,VBWA2,RHBA1
      4       ,RHBA2,CHBA1,CHBA2,FBWAV1,FBWAV2
 1000 FORMAT(I6,F8.5)
 1001 FORMAT(9F8.5)
 1002 FORMAT(////    )
 1003 FORMAT(1H1,42X33HTWO DIMENSIONAL HYDROFOIL FLUTTER //   )
 1004 FORMAT(33X3HD =F6.3,10X7HA      =F8.3,10X4HXA =F6.3 /33X3HF =F6.3,
     1      10X7HMU      =F8.3,10X4HGA =F6.3 /33X3HK =F6.3,10X7HRA**2 =
     2      F8.3,10X4HGH =F6.3 // )
 1005 FORMAT(41X38HDISCRIMINANT IS NEGATIVE - NO SOLUTION /// )
 1006 FORMAT(21X12H(WA/W)1**2 =F12.3, 5X12H(WA/W)2**2 =F12.3, 2X1H-, 2X
     1      11HNO SOLUTION /// )
 1007 FORMAT(21X12H(WA/W)2**2 =F12.3, 5X48H(WA/W)1**2 IS POSITIVE - NO S
     1OLUTION FOR (WA/W)2  // )
 1008 FORMAT(33X12H(WH/W)1**2 =F12.3,5X23HNO SOLUTION FOR (WH/W)1 // )
 1009 FORMAT(20X11H(WA/W)1    =F12.3  /20X11H(WH/WA)1   =F12.3 /20X
     1      11H(V/BWA)1   =F12.3/20X11H(FBWA/V)1 =F12.3/20X11HR(HO/BA)1 =
     2      F12.3 /20X11HI(HO/BA)1 =F12.3 )
 1010 FORMAT(33X12H(WH/W)2**2 =F12.3,5X23HNO SOLUTION FOR (WH/W)2 // )
 1011 FORMAT(20X11H(WA/W)1    =F12.3,30X11H(WA/W)2    =F12.3 /20X
     1      11H(WH/WA)1   =F12.3,30X11H(WH/WA)2   =F12.3/20X11H(V/BWA)1   =
     2      F12.3,30X11H(V/BWA)2   =F12.3/20X11H(FBWA/V)1 =F12.3,30X
     3      11H(FBWA/V)2 =F12.3/20X11HR(HO/BA)1 =F12.3,30X11HR(HO/BA)2 =
     4      F12.3/20X11HI(HO/BA)1 =F12.3,30X11HI(HO/BA)2 =F12.3 )
 1012 FORMAT(48X24HAERODYNAMIC COEFFICIENTS //30X4HL1 =F8.3,5X4HL2 =F8.3
     1       ,5X4HL3 =F8.3,5X4HL4 =F8.3 /30X4HM1 =F8.3,5X4HM2 =F8.3,
     2       5X4HM3 =F8.3,5X4HM4 =F8.3 //   )
    2 READ 1000,NC,A
      READ 1001,D,F,AK
    3 READ 1001,R1L,XI1L,R2L,XI2L,R1M,XI1M,R2M,XI2M
    4 XL1 = 1.0+2.0*XI1L/AK
      XL2 = -2.0*R1L/AK
      XL3 = -A+(XI2L-2.0*A*XI1L)/AK-2.0*R2L/AK**2
      XL4 = -(1.0+R2L-2.0*A*R1L)/AK-2.0*XI2L/AK**2
    5 XM1 = -A-(XI1M+2.0*A*XI1L)/AK
      XM2 = (R1M+2.0*A*R1L)/AK
      XM3 = .125+A**2+(A*(XI1M-XI2L)-.5*(XI2M-4.*A**2*XI1L))/AK+(R2M+2.*
     1      A*R2L)/AK**2
    6 XM4 = (A*(1.0+R2L -R1M)-.5*(1.0-R2M)-2.0*A**2*R1L)/AK+(XI2M+2.0*A*
     1       XI2L)/AK**2
      DO 7 I=1,NC
      READ 1001,AAMU(I),XXA(I),RRA(I),GGA(I),GGH(I)
    7 CONTINUE
      DO 40 I=1,NC
      AMU = AAMU(I)
      XA=XXA(I)
    8 RA = RRA(I)
      GA = GGA(I)
```

```
      GH = GGH(I)
    9 V1 = GH*AMU-XL2
      V2 = GA*AMU*RA-XM4
   10 AK1 = V2-GH*XM3
      AK2 = XL1*V2+XM3*V1+AMU*XA*(XM2+XL4)+XL3*XM2+XM1*XL4
      AK3 = GA+GH
      AK4 = GA*XL1-V1
   11 C1 = -(XL1+GA*V1)
      C2 = -XL1*XM3+V1*V2+AMU*XA*(AMU*XA+XL3+XM1)+XL3*XM1-XM2*XL4
      C3 = 1.0-GA*GH
      C4 = XM3+GH*V2
   12 B0 = C1*AK3+C3*AK4
      B1 = C2*AK3+C4*AK4-C1*AK1-C3*AK2
      B2 = -(C2*AK1+C4*AK2)
      IF(B0)120,13,13
  120 B0 = -B0
      B1 = -B1
      B2 = -B2
   13 DISC = B1**2-4.0*B0*B2
      IF(LL)14,15,14
   14 WRITE            (6   ,1002                                         )
      GO TO 16
   15 WRITE            (6   ,1003                                         )
   16 WRITE            (6   ,1004)D,A,XA,F,AMU,GA,AK,RA,GH
  160 WRITE            (6   ,1012)XL1,XL2,XL3,XL4,XM1,XM2,XM3,XM4
   17 LL = LL+1
      IF(LL-3)19,18,18
   18 LL = 0
   19 IF(DISC)20,21,21
   20 WRITE            (6   ,1005                                         )
      GO TO 40
   21 SQD = SQRT (DISC)
      SIG1 = (-B1-SQD)/(2.0*B0)
      SIG2 = (-B1+SQD)/(2.0*B0)
      SIH1 = (SIG1*C1+C2)/(SIG1*C3+C4)
      SIH2 = (SIG2*C1+C2)/(SIG2*C3+C4)
   22 WA12 = 1.0-SIG1/(AMU*RA)
      WA22 = 1.0-SIG2/(AMU*RA)
      WH12 = 1.0-SIH1/AMU
      WH22 = 1.0-SIH2/AMU
   23 IF(WA12)24,24,25
   24 WRITE            (6   ,1006)WA12,WA22
      GO TO 40
   25 WAW1 = SQRT (WA12)
      IF(WA22)26,26,260
   26 WRITE            (6   ,1007)WA22
  260 IF(WH12)27,28,28
   27 WRITE            (6   ,1008)WH12
      WHWA1 = 99999.0
      GO TO 29
   28 WHWA1 = SQRT (WH12)/WAW1
   29 VBWA1 = 1.0/(AK*WAW1)
      FBWAV1 = F*AK*WAW1
      RHBA1 = (AMU*XA+XM1)*(AMU*RA*(WA12-1.0)-XM3)+XM2*(WA12*GA-XM4)
      CHBA1 = (AMU*XA+XM1)*(WA12*GA-XM4)-XM2*(AMU*RA*(WA12-1.0)-XM3)
   30 IF(WA22)31,31,32
```

```
31 WRITE              (6   ,1009)WAW1,WHWA1,VBWA1,FBWAV1,RHBA1,CHBA1
   GO TO 40
32 WAW2 = SQRT (WA22)
   IF(WH22)33,34,34
33 WRITE              (6   ,1010)WH22
   WHWA2 = 99999.0
   GO TO 35
34 WHWA2 = SQRT (WH22)/WAW2
35 VBWA2 = 1.0/(AK*WAW2)
   FBWAV2 = F*AK*WAW2
   RHBA2 = (AMU*XA+XM1)*(AMU*RA*(WA22-1.0)-XM3)+XM2*(WA22*GA-XM4)
   CHBA2 = (AMU*XA+XM1)*(WA22*GA-XM4)-XM2*(AMU*RA*(WA22-1.0)-XM3)
36 WRITE              (6   ,1011)WAW1,WAW2,WHWA1,WHWA2,VBWA1,VBWA2,
  1     FBWAV1,FBWAV2,RHBA1,RHBA2,CHBA1,CHBA2
40 CONTINUE
   GO TO 2
   END
```

(B) "Hydrofoil Flutter Program . . . 1 Degree of Freedom - Pitching"

Written by H. Selib at the Cornell Aeronautical Laboratory, Inc.

The purpose of this IBM 7044 Fortran IV program is to solve Equations (8a) and (8b) of Section 2.3.2 for ($\frac{\omega_\alpha}{\omega}$ and a) once the values of D, F, k, & μr_α^2 have been prescribed. The specification of D, F, k is sufficient to determine the hydrodynamic functions H_{1L}, H_{2L}, and H_{1M}, H_{2M}, defined in Reference 3, and involved in Equations (8a) and (8b).

In the following the main program of length $_8$ 1013 (number of octal locations) was written in Fortran IV. The subroutine clear of length$_8$ 27, used to zero common storage, was not written in Fortran IV but is a "MAP" program.

The inputs are as follows:

CARD	FORMAT	
1	9F8.5	μr_α^2
2	9F8.5	D, F, K. If D = 99999999 start with a new value for μr_α^2
3	9F8.5	RH_{1L}, IH_{1L}; RH_{2L}, IH_{2L}; RH_{1M}, IH_{1M}, RH_{2M}, IH_{2M}.

The program print out for the single degree of freedom system consists of values of a, $\frac{\omega_\alpha}{\omega}$, $\omega_\alpha\sqrt{\frac{b}{g}}$, & $\frac{U}{b\omega_\alpha}$ as functions of D, F, k and μr_α^2.

The program listing follows on the next page.

```
$IBFTC ZZIFLT  LIST,REF
C       HYDROFOIL FLUTTER PROGRAM - ONE DEGREE OF FREEDOM
        COMMON AMURA2,NLIN,D,F,AK,F1L,G1L,F2L,G2L,F1M,G1M,F2M,G2M,B,C,
       1       DISC,IFLG,JFLG,KFLG,A1,A2,WA12,WA22,WBG1,WBG2,UBW1,UBW2,
       2       WA1,WA2
        EQUIVALENCE (D,ND)
     1  READ 1000,AMURA2
  1000  FORMAT (9G8.5)
        NLIN = 0
     2  CALL CLEAR(D,WA2)
        READ 1000,D,F,AK
        IF(ND.EQ.99999999) GO TO 1
        READ 1000,F1L,G1L,F2L,G2L,F1M,G1M,F2M,G2M
     3  B = (1.0+F2L-F1M+2.0*G2L/AK)/(4.0*F1L)
        C = (F2M+2.0*G2M/AK-1.0)/(4.0*F1L)
        DISC = B**2+C
        IF(DISC.GT.0.0) GO TO 4
        IFLG = 2
        GO TO 9
     4  IFLG = 1
        A1 = B+SQRT(DISC)
        A2 = B-SQRT(DISC)
        WA12 = 1.0+(A1**2*(1.0+2.0*G1L/AK)+A1*((G1M-G2L)/AK+2.0*F2L/AK**2)
       1       +.125-.5*G2M/AK+F2M/AK**2)/AMURA2
        WA22 = 1.0+(A2**2*(1.0+2.0*G1L/AK)+A2*((G1M-G2L)/AK+2.0*F2L/AK**2)
       1       +.125-.5*G2M/AK+F2M/AK**2)/AMURA2
     5  IF(WA12.GT.0.0) GO TO 6
        JFLG = 2
        WA1 = -SQRT(-WA12)
        GO TO 7
     6  JFLG = 1
        WA1 = SQRT(WA12)
        WBG1 = WA1*AK*F
        UBW1 = 1.0/(AK*WA1)
     7  IF(WA22.GT.0.0) GO TO 8
        KFLG = 2
        WA2 = -SQRT(-WA22)
        GO TO 9
     8  KFLG = 1
        WA2 = SQRT(WA22)
        WBG2 = WA2*AK*F
        UBW2 = 1.0/(AK*WA2)
     9  IF(NLIN.EQ.0) WRITE(6,1001)AMURA2
  1001  FORMAT(1H1,52X26HHYDROFOIL FLUTTER IN PITCH //53X14HMU*RALPHA**2 =
       1       E12.5 //14X1HD,10X1HF,10X1HK,14X1HA,14X4HWA/W, 8X
       2       12HWA*SQRT(B/G), 7X6HU/B*WA,10X6HB**2+C )
        GO TO (11,10),IFLG
    10  WRITE(6,1002)D,F,AK,DISC
  1002  FORMAT(F17.2,F13.5,F10.5,F81.5)
        NLIN = NLIN+1
        GO TO 17
    11  GO TO (12,13),JFLG
    12  WRITE(6,1003)D,F,AK,A1,WA1,WBG1,UBW1,DISC
  1003  FORMAT(F17.2,F13.5,F10.5,F15.5,F16.5,F18.5,F15.5,F17.5)
        NLIN = NLIN+1
        GO TO 14
```

```
   13 WRITE(6,1004)D,F,AK,A1,WA1,DISC
 1004 FORMAT(F17.2,F13.5,F10.5,F15.5,F16.5,F50.5)
      NLIN = NLIN+1
   14 GO TO (15,16),KFLG
   15 WRITE(6,1005) A2,WA2,WBG2,UBW2
 1005 FORMAT(F55.5,F16.5,F18.5,F15.5)
      NLIN = NLIN+1
      GO TO 17
   16 WRITE(6,1005) A2,WA2
      NLIN = NLIN+1
   17 IF(NLIN.GE.50)NLIN = 0
      GO TO 2
      END
```